A number of experimental and theoretical studies suggest that the presence of a modest radial shear in the axial plasma flow velocity can provide stabilization of Z-pinch plasmas against the most destructive kink and sausage ideal MHD instabilities, thereby making the flow stabilized Z-pinch (FSZP) systems attractive for magnetic fusion energy applications [1]. While radial variations in the plasma flow velocity that occur on the pinch-size scale $a$ can indeed generate sufficient phase-mixing for the stabilization of large-scale ($k \sim 1/a$) MHD modes, weaker short-scale drift-wave instabilities that occur on a much smaller ion gyroradius scale are less affected by the large-scale velocity shear. Left behind, the drift microturbulence can substantially enhance classical (collisional) transport coefficient including plasma viscosity and can act over time to reduce the velocity shear and degrade the confinement. In this paper we present the initial results from gyrokinetic simulations aimed to assess the influence of the ion gyro-scale drift microturbulence on the stability and transport properties of FSZP systems. The simulations are performed with the high-order finite-volume Eulerian gyrokinetic code COGENT and the results are analyzed for the parameters characteristic of the current and planned FSZP experiments.